arXiv:hep-ph/9910289v1 10 Oct 1999

Derivation of the Gasser-Leutwyler Lagrangian from QCD

Qing Wang[a,b], Yu-Ping Kuang[b,a], Xue-Lei Wang[a,c], Ming Xiao[a]

[a] *Department of Physics, Tsinghua University, Beijing 100084, China* *

[b] *China Center of Advanced Science and Technology (World Laboratory), P.O.Box 8730, Beijing 100080, China*

[c] *Department of Physics, Henan Normal University, Xinxiang 453002, China*

(TU-HEP-TH-99/105)

The normal part of the Gasser-Leutwyler formulation of the chiral Lagrangian is formally derived from the first principles of QCD without taking approximations. All the coefficients are expressed in terms of certain Green's functions in QCD, which can be regarded as the fundamental QCD definitions of the normal part of the coefficients. The approximate values of the claculated coefficients are also presented.

PACS number(s): 12.39.Fe, 11.30.Rd, 12.38.Aw, 12.38.Lg

Chiral Lagrangian (CL) [1] [2], especially the three-flavor formulation given by Gasser and Leutwyler [2], has been widely used in the study of low energy hadron physics. Studies on understanding the relation between the CL and the underlying theory of QCD will be very helpful for making the theory more predictive. There are papers studying approximate formulae for the CL [3–5] either based on certain assumptions or considering the anomaly contributions with certain approximations from the beginning. Further improvements are certainly needed. Actually, the study can be divided into two steps: (i) formally deriving the CL from QCD giving the fundamental QCD definitions of the coefficients, (ii) calculating the the values of the coefficients from the QCD definitions with certain approximations. This paper is mainly devoted to step (i) in which we derive the fundamental QCD formulae from the normal part contributions of the theory without taking approximations, which is different from and complimentary to the anomaly part in Ref. [5]. We shall also present the values of our approximately calculated coefficients [step (ii)].

Let A^i_μ be the gluon field, $\psi^{a\eta}_\alpha$ and $\Psi^{\bar{a}\eta}_\alpha$ be, respectively, the light and heavy quark fields with color index α, Lorentz spinor index η, light flavor index a and heavy flavor index $\bar{a}$. Following Gasser and Leutwyler, we start from the generating functional [2]

$$Z[J]=\int\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\Psi\mathcal{D}\bar{\Psi}\mathcal{D}A_\mu e^{i\int dx\{\mathcal{L}(\psi,\bar{\psi},\Psi,\bar{\Psi},A_\mu)+\bar{\psi}J\psi\}},\quad (1)$$

where $\mathcal{L}$ is the QCD Lagrangian with the gauge-fixing term and the Fadeev-Popov determinant. The local external sources $J_{\sigma\rho}$ can be decomposed as

$$J(x)=-s(x)+ip(x)\gamma_5+\not v\,(x)+\not a\,(x)\gamma_5,\qquad (2)$$

in which the light quark masses have been absorbed into s. Since the contributions from the anomaly to the CL has been studied in Ref. [5], our aim is to study the complete normal part contributions, and thus we simply take the θ-vacuum parameter $\theta=0$.

What we need to do is to *consistently* extract the pseudoscalar degree of freedom from the local composite operator $\bar{\psi}^{b\zeta}(x)\psi^{a\eta}(x)$ and integrate out all the remaining degrees of freedom. For this purpose, we consider the scalar and pseudoscalar degrees of freedom of $\bar{\psi}^{b\zeta}(x)\psi^{a\eta}(x)$ and make the following decomposition

$$\begin{aligned}\bar{\psi}^{b\zeta}(x)(1)_{\zeta\eta}\psi^{a\eta}(x)&=(\Omega'\sigma\Omega'+\Omega'^\dagger\sigma\Omega'^\dagger)^{ab}(x)\\ \bar{\psi}^{b\zeta}(x)(\gamma_5)_{\zeta\eta}\psi^{a\eta}(x)&=(\Omega'\sigma\Omega'-\Omega'^\dagger\sigma\Omega'^\dagger)^{ab}(x),\end{aligned}\qquad (3)$$

where the modular degree of freedom is described by an hermitian field $\sigma(x)$ $[\sigma^\dagger(x)=\sigma(x)]$, and the phase degree of freedom is described by a unitary field Ω' $[\Omega'^\dagger(x)\Omega'(x)=1]$. As usual, we can define $U'(x)\equiv\Omega'^2(x)$ which contains a $U(1)$ factor such that $\det U'(x)=e^{i\vartheta'(x)}$ ($\vartheta'(x)$ is real). We can extract out the $U(1)$ factor and define a field $U(x)$ as $U'(x)\equiv e^{\frac{i}{N_f}\vartheta'(x)}U(x)$ with $\det U(x)=1$. Then we can define a new field Ω and decompose U into

$$U(x)=\Omega^2(x),\qquad (4)$$

as in the literature. This $U(x)$, as the desired representation of $SU(N_f)_R\times SU(N_f)_L$, will be the nonlinear realization of the meson field in the CL. It is straightforward to eliminate σ from the two equations in (3) and get the following relations,

$$e^{-i\frac{\vartheta'}{N_f}}\Omega^\dagger\mathrm{tr}_l[P_R\psi\bar{\psi}]\Omega^\dagger=e^{i\frac{\vartheta'}{N_f}}\Omega\mathrm{tr}_l[P_L\psi\bar{\psi}]\Omega,\qquad (5)$$

$$e^{2i\vartheta'}=\det[\mathrm{tr}_l[P_R\psi\bar{\psi}]]/\det[\mathrm{tr}_l[P_L\psi\bar{\psi}]],\qquad (6)$$

in which all the fields are at the same space-time point x, and tr_l is the trace for the spinor index. Eqs.(3)-(6), especially (5), describe our idea of localization in the operator formalism.

To realize this idea in the functional integration, we need a technique to *integrate in* this information to (1). For this purpose, we start from the following functional identity for an operator $\mathcal{O}$ satisfying $\det\mathcal{O}=\det\mathcal{O}^\dagger$ [6],

$$\int\mathcal{D}U\ \delta(U^\dagger U-1)\delta(\det U-1)\ \mathcal{F}[\mathcal{O}]\ \delta(\Omega\mathcal{O}^\dagger\Omega-\Omega^\dagger\mathcal{O}\Omega^\dagger)=\mathrm{const}\,,\qquad (7)$$

in which $\int \mathcal{D}U\ \delta(U^\dagger U-1)\delta(\det U-1)$ is an invariant integration measure and the function $\mathcal{F}[\mathcal{O}]$ is defined as

$$\frac{1}{\mathcal{F}[\mathcal{O}]}\equiv \det\mathcal{O}\ \int \mathcal{D}\sigma\ \delta(\mathcal{O}^\dagger\mathcal{O}-\sigma^\dagger\sigma)\delta(\sigma-\sigma^\dagger)\,. \qquad (8)$$

With

$$\mathcal{O}(x)=e^{-i\frac{\vartheta'(x)}{N_f}}\mathrm{tr}_l[P_R\psi(x)\bar\psi(x)], \qquad (9)$$

(7) serves as the functional expression reflecting (5).

Inserting (7) and (9) into (1), we obtain

$$\begin{aligned}Z[J]&=\int \mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}\Psi\mathcal{D}\bar\Psi\mathcal{D}A_\mu\mathcal{D}U\ \delta(U^\dagger U-1)\delta(\det U-1)\\ &\times\delta\Big(e^{i\frac{\vartheta'}{N_f}}\Omega\mathrm{tr}_l[\psi_L\bar\psi_R]\Omega-e^{-i\frac{\vartheta'}{N_f}}\Omega^\dagger\mathrm{tr}_l[\psi_R\bar\psi_L]\Omega^\dagger\Big)\\ &\times\exp\big\{i\Gamma_I[\frac{\bar\psi\psi}{N_c}]+i\int d^4x\{\mathcal{L}(\psi,\bar\psi,\Psi,\bar\Psi,A_\mu)+\bar\psi J\psi\}\big\}\\ &=\int\mathcal{D}U\ \delta(U^\dagger U-1)\delta(\det U-1)\ e^{iS_{eff}[U,J]}, \qquad (10)\end{aligned}$$

where

$$\begin{aligned}e^{-i\Gamma_I[\frac{1}{N_c}\bar\psi\psi]}&=\prod_x\frac{1}{\mathcal{F}[\mathcal{O}(x)]}\\ &=\prod_x\big\{\big[\det\{\frac{1}{N_c}\mathrm{tr}_{lc}[\psi_R\bar\psi_L]\}\det\{\frac{1}{N_c}\mathrm{tr}_{lc}[\psi_L\bar\psi_R]\}\big]^{\frac12}\int\mathcal{D}\sigma\\ &\times\delta\big(\frac{1}{N_c^2}\mathrm{tr}_{lc}(\psi_R\bar\psi_L)\mathrm{tr}_{lc}(\psi_L\bar\psi_R)-\sigma^\dagger\sigma\big)\delta(\sigma-\sigma^\dagger)\big\}, \quad (11)\end{aligned}$$

and we have introduced the effective action

$$\begin{aligned}e^{iS_{eff}[U,J]}&=\int\mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}\Psi\mathcal{D}\bar\Psi\mathcal{D}A_\mu\delta(e^{i\frac{\vartheta'}{N_f}}\Omega\mathrm{tr}_l[\psi_L\bar\psi_R]\Omega\\ &-e^{-i\frac{\vartheta'}{N_f}}\Omega^\dagger\mathrm{tr}_l[\psi_R\bar\psi_L]\Omega^\dagger)\exp\Big\{i\Gamma_I[\frac{\bar\psi\psi}{N_c}]\\ &+i\int d^4x\{\mathcal{L}(\psi,\overline{\psi},\Psi,\overline{\Psi},A_\mu)+\overline{\psi}J\psi\}\Big\} \qquad (12)\end{aligned}$$

to formally express the integration over the quark and gluon fields.

To work out the $U(x)$-dependence of S_{eff}, we make the following chiral rotation

$$\begin{aligned}J_\Omega(x)&=[\Omega(x)P_R+\Omega^\dagger(x)P_L][J(x)+i\partial\!\!\!/]\\ &\quad\times[\Omega(x)P_R+\Omega^\dagger(x)P_L],\\ \psi_\Omega(x)&=[\Omega^\dagger(x)P_R+\Omega(x)P_L]\psi(x),\\ \bar\psi_\Omega(x)&=\bar\psi(x)[\Omega^\dagger(x)P_R+\Omega(x)P_L]. \qquad (13)\end{aligned}$$

Here we have denoted the rotated quantities by a subscript Ω. The fields U,A_μ,Ψ and $\bar\Psi$ are unchanged under the rotation. It is easiy to see from (6) that $\vartheta'_\Omega=\vartheta'$. In terms of the rotated fields, the δ-function in (12) is $\delta\Big(e^{i\frac{\vartheta'}{N_f}}\mathrm{tr}_l[\psi_{\Omega,L}\bar\psi_{\Omega,R}]-e^{-i\frac{\vartheta'}{N_f}}\mathrm{tr}_l[\psi_{\Omega,R}\bar\psi_{\Omega,L}]\Big)$, so that the explicit U-dependence of the theory is from the source term and the δ-function in (12). The remaining part in (12) is invariant under rotation (13) since the Lagrangian $\mathcal{L}$, together with the source-term, is chirally invariant, and from (13) we can see that $\Gamma\big[\frac{\bar\psi\psi}{N_c}\big]=\Gamma\big[\frac{\bar\psi_\Omega\psi_\Omega}{N_c}\big]$. Since the integration measure $\mathcal{D}\psi\mathcal{D}\bar\psi$ is not invariant under the chiral rotation, it will cause certain anomaly terms in S_{eff} i.e. $\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}A_\mu=\int\mathcal{D}\psi_\Omega\mathcal{D}\overline{\psi}_\Omega\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}A_\mu\ e^{[\text{anomaly terms}]}$. Thus, making a chiral rotation of ψ and $\bar\psi$, we have

$$\begin{aligned}e^{iS_{\rm eff}[1,J_\Omega]}&=\int\mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}\Psi\mathcal{D}\bar\Psi\mathcal{D}A_\mu\ \delta\Big(e^{i\frac{\vartheta'}{N_f}}\mathrm{tr}_l[\psi_L\bar\psi_R]\\ &\quad-e^{-i\frac{\vartheta'}{N_f}}\mathrm{tr}_l[\psi_R\bar\psi_L]\Big)\exp\big\{i\Gamma_I[\frac{\bar\psi\psi}{N_c}]\\ &\quad+i\int d^4x[\mathcal{L}(\psi,\bar\psi,\Psi,\bar\Psi,A_\mu)+\bar\psi J_\Omega\psi]+\text{anomaly terms}\big\}\\ &=\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\Psi\mathcal{D}\bar\Psi\mathcal{D}A_\mu\ \delta\Big(\bar\psi^a(-i\sin\frac{\vartheta'}{N_f}+\gamma_5\cos\frac{\vartheta'}{N_f})\psi^b\Big)\\ &\quad\times\exp\big\{i\Gamma_I[\frac{\bar\psi\psi}{N_c}]+i\int d^4x[\mathcal{L}(\psi,\overline{\psi},\Psi,\bar\Psi,A_\mu)+\bar\psi(v\!\!\!/_\Omega\\ &\quad+a\!\!\!/_\Omega\gamma_5-s_\Omega-p_\Omega\tan\frac{\vartheta'}{N_f})\psi]+\text{anomaly terms}\big\}. \qquad (14)\end{aligned}$$

In (14) and all the later formulae, we simply use the symbol ψ ($\bar\psi$) as the short notations of ψ_Ω ($\bar\psi_\Omega$) for shortening the formulae. This does not make confusion since they are integration variables). We see that, in (14), the U-dependence of S_{eff} comes only from the rotated sources. So that the rotation simplifies the U-dependence of S_{eff}. Eq.(14) shows that $S_{eff}[1,J_\Omega]$ is the QCD generating functional for the rotated sources with the $-i\bar\psi^a\psi^b\sin\frac{\vartheta'}{N_f}$ $+\bar\psi^a\gamma_5\psi^b\cos\frac{\vartheta'}{N_f}$ degree of freedom frozen. After making a further $U_A(1)$ rotation of the ψ and $\bar\psi$, $\frac{\vartheta'}{N_f}$ can be rotated away and the frozen degree of freedom just becomes the pesudoscalar degree of freedom $\bar\psi^a\gamma_5\psi^b$ as it should be since this degree of freedom is already included in the U-field. The automatic occurance of this frozen degree of freedom implies that our way of extracting the U-field degree of freedom is really *consistent*.

We first consider the p^2-order terms. Expanding (14) up to the order of p^2, and taking account of translational invariance, parity and flavor conservations, we obtain

$$\begin{aligned}S_{eff}[1,J_\Omega]\Big|_{O(p^2)}&=F_0^2\int dx\ \mathrm{tr}[a_\Omega^2+B_0s_\Omega]\\ &=\frac{F_0^2}{4}\int dx\ \mathrm{tr}\Big[[\nabla^\mu U^\dagger][\nabla_\mu U]+[U\chi^\dagger+U^\dagger\chi]\Big], \qquad (15)\end{aligned}$$

where ∇_μ is the covariant derivative related to the external sources defined in Ref. [2], $\chi\equiv 2B_0(s+ip)$, and the coefficients F_0^2 and B_0 are defined in terms of the following QCD Green's functions as

$$F_0^2B_0\equiv-\frac{1}{N_f}\Big\langle\bar\psi\psi\Big\rangle, \qquad (16)$$

and

$$F_0^2 \equiv \frac{i}{8(N_f^2-1)}\int dx\Big[\Big\langle \overline{\psi}^a(0)\gamma^\mu\gamma_5\psi^b(0)\bar{\psi}^b(x)\gamma_\mu\gamma_5\psi^a(x)\Big\rangle_C$$
$$-\frac{1}{N_f}\Big\langle[\overline{\psi}^a(0)\gamma^\mu\gamma_5\psi^a(0)][\overline{\psi}^b(x)\gamma_\mu\gamma_5\psi^b(x)]\Big\rangle_C\Big]. \qquad (17)$$

In (16) and (17), the vacuum expectation values are defined as, for an operator O,

$$\Big\langle O\Big\rangle \equiv \frac{\int \mathcal{D}\mu\ O}{\int\mathcal{D}\mu},$$
$$\mathcal{D}\mu \equiv \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\Psi\mathcal{D}\bar{\Psi}\mathcal{D}A_\mu\delta\Big(\bar{\psi}^a\big(-i\sin\frac{\vartheta'}{N_f}+\gamma_5\cos\frac{\vartheta'}{N_f}\big)\psi^b\Big)$$
$$\times e^{i\Gamma_I[\frac{\bar{\psi}\psi}{N_c}]+i\int dx\mathcal{L}(\psi,\bar{\psi},\Psi,\bar{\Psi},A_\mu)}, \qquad (18)$$

and $\langle\cdots\rangle_C$ denotes the connected part of $\langle\cdots\rangle$. The integrand in (15) is just the p^2-order terms in the Gasser-Leutwyler Lagrangian [2].

The p^4-order terms can be worked out along the same line. With the help of the p^2-order equation of motion, we obtain the normal part contributions (ignoring anomaly contributions) to the p^4-order terms,

$$S_{eff}[1,J_\Omega]\Big|_{O(p^4)} = \int dx\ \mathrm{tr}\Big[-\mathcal{K}_1[d_\mu a_\Omega^\mu]^2$$
$$-\mathcal{K}_2(d^\mu a_\Omega^\nu - d^\nu a_\Omega^\mu)(d_\mu a_{\Omega,\nu}-d_\nu a_{\Omega,\mu}) + \mathcal{K}_3[a_\Omega^2]^2$$
$$+\mathcal{K}_4 a_\Omega^\mu a_\Omega^\nu a_{\Omega,\mu}a_{\Omega,\nu} + \mathcal{K}_5 a_\Omega^2\mathrm{tr}[a_\Omega^2] + \mathcal{K}_6 a_\Omega^\mu a_\Omega^\nu \mathrm{tr}[a_{\Omega,\mu}a_{\Omega,\nu}]$$
$$+\mathcal{K}_7 s_\Omega^2 + \mathcal{K}_8 s_\Omega\mathrm{tr}[s_\Omega] + \mathcal{K}_9 p_\Omega^2 + \mathcal{K}_{10}p_\Omega\mathrm{tr}[p_\Omega] + \mathcal{K}_{11}s_\Omega a_\Omega^2$$
$$+\mathcal{K}_{12}s_\Omega\mathrm{tr}[a_\Omega^2] - \mathcal{K}_{13}V_\Omega^{\mu\nu}V_{\Omega,\mu\nu} + i\mathcal{K}_{14}V_\Omega^{\mu\nu}a_{\Omega,\mu}a_{\Omega,\nu}$$
$$+\mathcal{K}_{15}p_\Omega d^\mu a_{\Omega,\mu}\Big]$$
$$= \int dx\Big[L_1[\mathrm{tr}(\nabla^\mu U^\dagger\nabla_\mu U)]^2$$
$$+L_2\mathrm{tr}[\nabla_\mu U^\dagger\nabla_\nu U]\mathrm{tr}[\nabla^\mu U^\dagger\nabla^\nu U]$$
$$+L_3\mathrm{tr}[(\nabla^\mu U^\dagger\nabla_\mu U)^2] + L_4\mathrm{tr}[\nabla^\mu U^\dagger\nabla_\mu U]\mathrm{tr}[\chi^\dagger U + U^\dagger\chi]$$
$$+L_5\mathrm{tr}[\nabla^\mu U^\dagger\nabla_\mu U(\chi^\dagger U+U^\dagger\chi)] + L_6[\mathrm{tr}(\chi^\dagger U+U^\dagger\chi)]^2$$
$$+L_7[\mathrm{tr}(\chi^\dagger U - U^\dagger\chi)]^2 + L_8\mathrm{tr}[\chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger]$$
$$-iL_9\mathrm{tr}[F_{\mu\nu}^R\nabla^\mu U\nabla^\nu U^\dagger + F_{\mu\nu}^L\nabla^\mu U^\dagger\nabla^\nu U]$$
$$+L_{10}\mathrm{tr}[U^\dagger F_{\mu\nu}^R U F^{L,\mu\nu}] + H_1\mathrm{tr}[F_{\mu\nu}^R F^{R,\mu\nu} + F_{\mu\nu}^L F^{L,\mu\nu}]$$
$$+H_2\mathrm{tr}[\chi^\dagger\chi]\Big], \qquad (19)$$

where $d^\mu a_\Omega^\nu \equiv \partial^\mu a_\Omega^\nu - iv_\Omega^\mu a_\Omega^\nu + ia_\Omega^\mu v_\Omega^\nu$, $V_\Omega^{\mu\nu} \equiv \partial^\mu v_\Omega^\nu - \partial^\nu v_\Omega^\mu - iv_\Omega^\mu v_\Omega^\nu + iv_\Omega^\nu v_\Omega^\mu$, and

$$L_1 = \frac{\mathcal{K}_4+2\mathcal{K}_5+2\mathcal{K}_{13}-\mathcal{K}_{14}}{32},$$
$$L_2 = \frac{\mathcal{K}_4+\mathcal{K}_6+2\mathcal{K}_{13}-\mathcal{K}_{14}}{16},$$
$$L_3 = \frac{\mathcal{K}_3-2\mathcal{K}_4-6\mathcal{K}_{13}+3\mathcal{K}_{14}}{16},\ L_4 = \frac{\mathcal{K}_{12}}{16B_0},$$
$$L_5 = \frac{\mathcal{K}_{11}}{16B_0},\ L_6 = \frac{\mathcal{K}_8}{16B_0^2},\ L_7 = -\frac{B_0^2\mathcal{K}_1+N_f\mathcal{K}_{10}+B_0\mathcal{K}_{15}}{16B_0^2N_f},$$
$$L_8 = \frac{B_0^2\mathcal{K}_1+\mathcal{K}_7-\mathcal{K}_9+B_0\mathcal{K}_{15}}{16B_0^2}, L_9 = \frac{4\mathcal{K}_{13}-\mathcal{K}_{14}}{8},$$
$$L_{10} = -\frac{\mathcal{K}_2+\mathcal{K}_{13}}{2},\ H_1 = \frac{\mathcal{K}_2-\mathcal{K}_{13}}{4},$$
$$H_2 = \frac{-B_0^2\mathcal{K}_1+\mathcal{K}_7+\mathcal{K}_9-B_0\mathcal{K}_{15}}{8B_0^2}. \qquad (20)$$

The coefficients $\mathcal{K}_1,\cdots\mathcal{K}_{15}$ are defined by the following integrations of the QCD Green's functions

$$\frac{i}{4}\int dx\ x^{\mu'}x^{\nu'}\Big\langle[\bar{\psi}^a(0)\gamma^\mu\gamma_5\psi^b(0)][\bar{\psi}^c(x)\gamma^\nu\gamma_5\psi^d(x)]\Big\rangle_C$$
$$= [(\frac{\mathcal{K}_1}{2}-\mathcal{K}_2)(g^{\mu\mu'}g^{\nu\nu'}+g^{\mu\nu'}g^{\nu\mu'}) + 2\mathcal{K}_2 g^{\mu\nu}g^{\mu'\nu'}]\delta^{ad}\delta^{bc}$$
$$+\cdots$$
$$-\frac{i}{24}\int dxdydz\Big\langle[\bar{\psi}^a(0)\gamma^\mu\gamma_5\psi^b(0)][\bar{\psi}^c(x)\gamma^\nu\gamma_5\psi^d(x)]$$
$$\times[\bar{\psi}^e(y)\gamma^\lambda\gamma_5\psi^f(y)][\bar{\psi}^g(z)\gamma^\kappa\gamma_5\psi^h(z)]\Big\rangle_C$$
$$= \frac{1}{6}\Big\{\delta^{ad}\Big[\delta^{cf}\delta^{eh}\delta^{gb}[\frac{1}{2}(g^{\mu\nu}g^{\lambda\kappa}+g^{\mu\kappa}g^{\nu\lambda})\mathcal{K}_3 + g^{\mu\lambda}g^{\nu\kappa}\mathcal{K}_4]$$
$$+\delta^{ch}\delta^{gf}\delta^{eb}[\frac{1}{2}(g^{\mu\nu}g^{\lambda\kappa}+g^{\mu\lambda}g^{\nu\kappa})\mathcal{K}_3 + g^{\mu\kappa}g^{\nu\lambda}\mathcal{K}_4]\Big]$$
$$+\delta^{af}\Big[\delta^{ed}\delta^{ch}\delta^{gb}[\frac{1}{2}(g^{\mu\lambda}g^{\nu\kappa}+g^{\mu\kappa}g^{\nu\lambda})\mathcal{K}_3 + g^{\mu\nu}g^{\lambda\kappa}\mathcal{K}_4]$$
$$+\delta^{eh}\delta^{gd}\delta^{cb}[\frac{1}{2}(g^{\mu\lambda}g^{\nu\kappa}+g^{\mu\nu}g^{\lambda\kappa})\mathcal{K}_3 + g^{\mu\kappa}g^{\nu\lambda}\mathcal{K}_4]\Big]$$
$$+\delta^{ah}\Big[\delta^{gd}\delta^{cf}\delta^{eb}[\frac{1}{2}(g^{\mu\kappa}g^{\nu\lambda}+g^{\mu\lambda}g^{\nu\kappa})\mathcal{K}_3 + g^{\mu\nu}g^{\lambda\kappa}\mathcal{K}_4]$$
$$+\delta^{gf}\delta^{ed}\delta^{cb}[\frac{1}{2}(g^{\mu\kappa}g^{\nu\lambda}+g^{\mu\nu}g^{\lambda\kappa})\mathcal{K}_3 + g^{\mu\lambda}g^{\nu\kappa}\mathcal{K}_4]\Big]$$
$$+\delta^{ad}\delta^{cb}\delta^{eh}\delta^{gf}[g^{\mu\nu}g^{\lambda\kappa}2\mathcal{K}_5 + (g^{\mu\lambda}g^{\nu\kappa}+g^{\mu\kappa}g^{\nu\lambda})\mathcal{K}_6]$$
$$+\delta^{af}\delta^{eb}\delta^{ch}\delta^{gd}[g^{\mu\lambda}g^{\nu\kappa}2\mathcal{K}_5 + (g^{\mu\nu}g^{\lambda\kappa}+g^{\mu\kappa}g^{\nu\lambda})\mathcal{K}_6]$$
$$+\delta^{ah}\delta^{gb}\delta^{cf}\delta^{ed}[g^{\mu\kappa}g^{\nu\lambda}2\mathcal{K}_5 + (g^{\mu\nu}g^{\lambda\kappa}+g^{\mu\lambda}g^{\nu\kappa})\mathcal{K}_6]\Big\} + \cdots$$
$$\frac{i}{2}\int dx\Big\langle[\bar{\psi}^a(0)\psi^b(0)][\bar{\psi}^c(x)\psi^d(x)]\Big\rangle_C$$
$$= \mathcal{K}_7\delta^{ad}\delta^{bc} + \mathcal{K}_8\delta^{ab}\delta^{cd}$$
$$\frac{i}{2}\int dx\Big\langle[\bar{\psi}^a(0)\psi^b(0)]\tan\frac{\vartheta'(0)}{N_f}[\bar{\psi}^c(x)\psi^d(x)]\tan\frac{\vartheta'(x)}{N_f}\Big\rangle_C$$
$$= \mathcal{K}_9\delta^{ad}\delta^{bc} + \mathcal{K}_{10}\delta^{ab}\delta^{cd}$$
$$\frac{1}{8}\int dxdy\Big\langle[\bar{\psi}^a(0)\psi^b(0)][\bar{\psi}^c(x)\gamma^\mu\gamma_5\psi^d(x)]$$
$$\times[\bar{\psi}^e(y)\gamma_\mu\gamma_5\psi^f(y)]\Big\rangle_C$$
$$= \frac{1}{2}\mathcal{K}_{11}(\delta^{ad}\delta^{cf}\delta^{eb}+\delta^{af}\delta^{ed}\delta^{cb}) + \mathcal{K}_{12}\delta^{ab}\delta^{cf}\delta^{ed} + \cdots,$$

$$\mathcal{K}_{13} = \frac{i}{576(N_f^2-1)}\int dx \Bigg[(5g_{\mu\nu}g_{\mu'\nu'} - 2g_{\mu\mu'}g_{\nu\nu'})x^{\mu'}x^{\nu'} \times[\Bigg\langle\ [\overline{\psi}^a(0)\gamma^\mu\psi^b(0)]\ [\bar{\psi}^b(x)\gamma^\nu\psi^a(x)]\ \Bigg\rangle_C - \frac{1}{N_f}\Bigg\langle\ [\bar{\psi}^a(0)\gamma^\mu\psi^a(0)][\overline{\psi}^b(x)\gamma^\nu\psi^b(x)]\ \Bigg\rangle_C]\Bigg]$$

$$\mathcal{K}_{14} = \frac{i}{36}2g_{\mu\kappa}g_{\nu\lambda}T_A^{\mu\nu\lambda\kappa} + 2g_{\mu\nu}g_{\lambda\kappa}T_A^{\mu\nu\lambda\kappa} - g_{\mu\lambda}g_{\nu\kappa}T_A^{\mu\nu\lambda\kappa} -2g_{\mu\kappa}g_{\nu\lambda}T_B^{\mu\nu\lambda\kappa} - 2g_{\mu\nu}g_{\lambda\kappa}T_B^{\mu\nu\lambda\kappa} + g_{\mu\lambda}g_{\nu\kappa}T_B^{\mu\nu\lambda\kappa}$$

$$\mathcal{K}_{15} = \frac{i}{4(N_f^2-1)}\int d^4x\ x^\mu\Bigg[\Bigg\langle\bar{\psi}^a(0)\psi^b(0)\tan\frac{\vartheta'(0)}{N_f} \times\bar{\psi}^b(x)\gamma_\mu\gamma_5\psi^a(x)\Bigg\rangle_C - \frac{1}{N_f}\Bigg\langle\bar{\psi}^a(0)\psi^a(0)\tan\frac{\vartheta'(0)}{N_f} \times\bar{\psi}^b(x)\gamma_\mu\gamma_5\psi^b(x)\Bigg\rangle_C\Bigg] \quad (21)$$

with $T_A,\ T_B$ defined as

$$-\frac{1}{2}\int dxdy\ x^\kappa\Bigg\langle\ [\overline{\psi}^a(0)\gamma^\mu\psi^b(0)][\overline{\psi}^c(x)\gamma^\nu\gamma_5\psi^d(x)] \times[\overline{\psi}^e(y)\gamma^\lambda\gamma_5\psi^f(y)]\Bigg\rangle_C = \delta^{ad}\delta^{cf}\delta^{eb}T_A^{\mu\nu\lambda\kappa} + \delta^{af}\delta^{ed}\delta^{cb}T_B^{\mu\nu\lambda\kappa} + \cdots, \quad (22)$$

in which the unwritten terms are those which do not contribute in (19). The integrand in (19) is just the p^4-order terms in the Gasser-Leutwyler Lagrangian.

Eqs.(16), (17), and (20)-(22) can be regarded as *the fundamental QCD definitions of the normal parts of* $F_0^2, B_0, L_1, \cdots, L_{10}, H_1$ *and* H_2. In principle, the related Green's functions can be calculated from lattice QCD.

As our first calculated result, we present here the values of the coefficients calculated from solving the equations for the related Green's functions in a crude approximation. We consider only the contributions from the two-point Green's functions, and take the approximations of the large-N_c limit and the leading order in dynamical perturbation theory [7,4]. In these approximations, our formula (17) for F_0^2 reduces to the well-known Pagels-Stokar formula [7]. In the present approximation, the QCD effective coupling constant is approximately $\alpha_s(p^2) \approx g_s^2/\big(4\pi[1+\Pi_G(p^2)]\big)$ [8], where Π_G is the gluon self-energy. In the evaluation of the Green's functions, we further take the approximation $\alpha_s[(p-q)^2] \approx \alpha_s[max(p^2,q^2)]$ to reduce the integral equations to differential equations. In the calculation, we take the modified one-loop formula for α_s, i.e. $\alpha_s(p^2) = \frac{12\pi}{(33-N_f)\ln(p^2/\Lambda_{QCD}^2+\tau)}$ [9]. Since renormalization schemes cannot be distinguished in the one-loop formula and we take $N_f = 3$, the value of Λ_{QCD} is not to be compared with the experimental value of $\Lambda_{\overline{MS}}^{(4)}$.

In the numerical calculations, we take $F_0 = 87$ MeV [10] as input by which τ is related to Λ_{QCD}. We present here, in Table 1, the numerical results of the complete coefficients $L_1, \cdots, L_{10}, H_1$ and H_2 (the normal part plus the anomaly contribution [5]) for $\Lambda_{QCD} = 600,\ 700,\ 800$ MeV (see Ref. [11] for details).

Table 1. Values of the complete p^4-order coefficients (in 10^{-3}) for various values of Λ_{QCD}, and the comparison with the values determined by experiments from Ref. [2].

Complete Coefficients	Λ_{QCD} (MeV) 600	700	800	Expt.
L_1	1.0	1.0	1.0	0.9± 0.3
L_2	2.1	2.1	2.1	1.7± 0.7
L_3	-3.1	-2.5	-2.0	-4.4± 2.5
L_4	0	0	0	0± 0.5
L_5	4.0	4.7	5.4	2.2± 0.5
L_6	0	0	0	0± 0.3
L_7	-0.27	-0.41	-0.50	-0.4± 0.15
L_8	0.006	0.44	0.69	1.1± 0.3
L_9	6.3	6.3	6.3	7.4± 0.7
L_{10}	-2.4	-2.1	-2.1	-6.0± 0.7
H_1	1.2	1.1	1.0	
H_2	-0.01	-0.89	-1.4	

We see from Table 1 that all results are of the right sign and most of them are very close to the experimental values except L_5, L_8 and L_{10}. Existing certain deviation is understandable since the present approximations are rather crude. Improvement of the approach is in progress.

This work is supported by the National Natural Science Foundation of China and the Fundamental Research Grant of Tsinghua University.

[1] S. Weinberg, Physica, **96A**, 327 (1979).

[2] J. Gasser and H. Leutwyler, Ann. Phys. **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).

[3] B. Holdom, Phys. Rev. D**45**, 2534 (1992); B. Holdom, J. Terning, and K. Verbeek, Phys. Lett. **B245**, 612 (1990); **B273**, 549(E) (1991).

[4] B. Holdom, *ibid.* **B294**, 293 (1992); B. Holdom and Q. Wang, Phys. Rev. D**55**, 3101 (1997).

[5] D. Espriu, E. De Rafael, and J. Taron, Nucl. Phys. **B345**, 22 (1990), and references therein.

[6] Q. Wang, Y.-P. Kuang, M. Xiao, and X.-L. Wang, Tsinghua Univ. Preprint TUIMP-TH-99/100, hep-ph/9903201.

[7] H. Pagels and S. Stokar, Phys. Rev. D**20**, 2947 (1979).

[8] M.R. Frank and C.D. Roberts, Phys. Rev. C **53**, 390 (1995).

[9] D. Atkinson and P.W. Johnson, Phys. Rev. D **37**, 2296 (1998); *ibid* D **41**, 1661 (1990).

[10] J. Gasser and H. Leutwyler, Nucl. Phys. B **250**, 517 (1995).

[11] Q. Wang, Y.-P. Kuang, M. Xiao, and X.-L. Wang, *Calculation of the Chiral Lagrangian Coefficients from the Underlying Theory of QCD* (in preparation).